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Interrupted coarsening in the zero-temperature kinetic Ising chain driven by a periodic external field

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Abstract

If quenched to zero temperature, the one-dimensional Ising spin chain undergoes coarsening, whereby the density of domain walls decays algebraically in time. We show that this coarsening process can be interrupted by exerting a rapidly oscillating periodic field with enough strength to compete with the spin-spin interaction. By analyzing correlation functions and the distribution of domain lengths both analytically and numerically, we observe nontrivial correlation with more than one length scale at the threshold field strength.

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I. INTRODUCTION

One of the most important topics in statistical physics is the formation of order. A classical nonequilibrium example is provided by the one-dimensional (1D) Glauber-Ising model quenched to a zero temperature. It approaches one of the ordered ground states by forming larger and larger domains [1], and this coarsening process has been analyzed in full detail (see, e.g., Ref. 2). The relaxation toward equilibrium is very slow: In the absence of an external field, the domain walls perform annihilating random walks and the density accordingly decays as $\rho \sim t^{-1/2}$ as time t goes by [3]. A kinetically constrained version also exhibits glassy behavior with anomalously slower coarsening [4–6], and such "aging" can even cease to proceed under steady driving [7]. One may ask if something similar can be achieved in the original Glauber-Ising system by driving it with a suitable protocol, as has been pointed out in Ref. 8. Evidently, a constant external field does not work for that purpose because the field will only accelerate the coarsening dynamics by breaking the up-down symmetry. If the symmetry is concerned, an alternative protocol would be an oscillating field with a short period. This is of particular interest from the perspective of interaction of light and matter in the high-frequency regime. The problem becomes highly nontrivial, especially when the matter has internal spatiotemporal correlations as a many-body system (see, e.g., Refs. 9–11 and references therein). Due to its ubiquity and often dramatic consequences as reported in Ref. 12, the nonequilibrium caused by oscillatory driving still remains as an active area of research to be explored further [13, 14]. If we consider the 1D Ising chain under an oscillating field, one possible scenario is that the system has such a large time scale that it simply overlooks the rapid oscillation so that the field appears as a small perturbation around the ordered state. Indeed, this has been numerically observed in the two-dimensional kinetic Ising model subjected to an oscillating field (see, e.g., Ref. 15). On the other hand, it also seems plausible that the disordered state can remain stable, although energetically unfavorable, just as an inverted pendulum is stabilized by fast oscillatory driving [16, 17]. In this paper, we show that the latter is the case when the field strength is greater than or equal to the spin-spin coupling strength. In fact, from the dynamic rules defined below, we can readily convince ourselves that all the correlations are completely destroyed if the field amplitude exceeds the spin-spin coupling strength, which effectively corresponds to an infinite temperature. If the field is weaker

than the spin-spin interaction, on the other hand, the up-down symmetry can be broken as in an ordered phase because the system cannot escape from the absorbing states with all the spins aligned in one direction. Only when the internal and external energy scales are equally strong, we observe finite nontrivial correlations and a stationary density of domain walls. We will explain this point by calculating correlation functions and the distribution of domain lengths both analytically and numerically.

This paper is organized as follows: An explanation of our model system is given in Sec. II. Correlation functions and the domain length distribution are analyzed in Sec. III. This is followed by a discussion of results and conclusions.

II. GLAUBER-ISING DYNAMICS

Let us consider a 1D Ising chain with size L under a time-dependent external field $H(t)$. The energy function is written as

$$E = -\frac{J}{2} \sum_{i=1}^{L} S_i S_{i+1} - H(t) \sum_{i=1}^{L} S_i, \tag{1}$$

where the spin variable S_i can take either $+1$ or -1 and $\frac{J}{2} > 0$ is the coupling strength between neighboring spins. We will impose a periodic boundary condition by setting $S_{L+1} = S_1$. The time evolution of this system is assumed to obey the zero-temperature Glauber dynamics [18], which means that every spin flips with the following rate:

$$W_i = \begin{cases} 1 & \text{if} \quad \Delta E_i < 0, \\ \frac{1}{2} & \text{if} \quad \Delta E_i = 0, \\ 0 & \text{if} \quad \Delta E_i > 0, \end{cases} \tag{2}$$

where ΔE_i is the energy difference due to a spin flip from S_i to $-S_i$. The external field $H(t)$ takes a rectangular pulse shape between $+H_0$ and $-H_0$ with period $2T$, where $H_0 > 0$ is a constant. It is convenient to define $\tau \equiv (t \mod 2T)$ as a time index within each period. Then, the external field is described as follows:

$$H(t) = \begin{cases} +H_0 & \text{for} \quad 0 \leqslant \tau < T, \\ -H_0 & \text{for} \quad T \leqslant \tau < 2T. \end{cases} \tag{3}$$

As briefly mentioned above, we need to consider competition between the spin-spin interaction and the external driving: If $H_0 > J$, the field direction solely determines the dynamics,

so that the system is equivalent to a collection of non-interacting spins subjected to the field. If $H_0 < J$, on the other hand, the field cannot flip a spin once it is surrounded by two other spins in the same direction. As a consequence, the density of domain walls keeps decreasing, regardless of the field direction, playing the role of the Lyapunov function in this dynamics. This means that the steady states under periodic driving must be the ordered ones for $H_0 < J$, and the deterministic nature of the dynamics suggests that the coarsening will not be slower than the field-free case. One can indeed numerically check that the density of domain walls decays as $\rho \sim t^{-1/2}$ when T is small but with a smaller prefactor than in the absence of $H(t)$. For this reason, we can say that $H_0 = J$ is the most nontrivial point due to the interplay between the field and the spin-spin interaction. Henceforth, we will set $H_0 = J$ unless otherwise mentioned.

As is well known, the dynamics can also be analyzed in terms of domain walls. We will briefly review three basic processes of the domain-wall dynamics, i.e., pair creation, pair annihilation, and propagation, assuming that $H(t) = +H_0$. First, two domain walls are created inside a down-spin domain $\{\cdots \downarrow\downarrow\downarrow \cdots\}$ when the field flips the spin in the middle with rate $\frac{1}{2}$, which results in $\{\cdots \downarrow\uparrow\downarrow \cdots\}$. Second, the pair-annihilation process is possible in two different ways, i.e., $\{\cdots \uparrow\downarrow\uparrow \cdots\} \Longrightarrow \{\cdots \uparrow\uparrow\uparrow \cdots\}$ with rate 1 or $\{\cdots \downarrow\uparrow\downarrow \cdots\} \Longrightarrow \{\cdots \downarrow\downarrow\downarrow \cdots\}$ with rate $\frac{1}{2}$. Last, a domain wall propagates when a spin flips at a domain boundary, e.g., $\{\cdots \downarrow\downarrow\uparrow \cdots\} \Longrightarrow \{\cdots \downarrow\uparrow\uparrow \cdots\}$ with rate 1. In the spin language, all these processes tend to align spins along the field direction. Therefore, few domain walls exist if the field has been applied for a sufficiently long period. One of our primary interests is how the density of domain walls varies in time when the time-dependent field in Eq. (3) drives the system.

We can formally describe the Glauber-Ising dynamics by using the transition-matrix formulation because it is Markovian. The Ising chain in Eq. (1) has $N = 2^L$ microstates. Indexing the microstates by $\alpha = 1, \ldots, N$, we define $p_\alpha(t)$ as the probability to find the system in state α at time t. The probability distribution can then be denoted as $\mathbf{p}(t) \equiv \{p_1(t), p_2(t), ..., p_N(t)\}$ with a constraint for the conservation of total probability, $\sum_\alpha p_\alpha(t) = 1$. One can readily calculate any single-time observable from $\mathbf{p}(t)$ in principle, including the average domain wall density. The zero-temperature Glauber rates in Eq. (2) define an $N \times N$

transition matrix $\mathbf{M}(t)$ that governs the evolution of $\mathbf{p}(t)$ in the following way:

$$\mathbf{p}(t + \Delta t) = \mathbf{M}(t)\mathbf{p}(t), \tag{4}$$

where Δt means a time scale for flipping a single spin. It is reasonable to suppose that every spin has a chance to flip during one time step on average, which means that Δt should be proportional to L^{-1}. The rates are dependent on the external field, so we can distinguish the rates under $H(t) = +H_0$ from those under $-H_0$. It implies that we have to work with two transition matrices:

$$\mathbf{M}(t) = \begin{cases} \mathbf{M}^{+} & \text{if} \quad H(t) = +H_0, \\ \mathbf{M}^{-} & \text{if} \quad H(t) = -H_0, \end{cases} \tag{5}$$

which are actually related by a simple coordinate transformation [14]. After one period, therefore, the probability distribution at time $t = 0$ evolves to $\mathbf{p}(t = 2T) = \mathbf{M}_T \mathbf{p}(t = 0)$ with $\mathbf{M}_T \equiv [(\mathbf{M}^{-})^{L}]^{T}[(\mathbf{M}^{+})^{L}]^{T}$. When the system has been entrained by the driving, it should be found statistically identical at time t and $t + 2T$. This can be regarded as a nonequilibrium steady state in a stroboscopic sense. For example, we may observe the system at the beginning of every period, i.e., at $\tau = 0$ and denote the resulting steady state as $\mathbf{p}_\infty(\tau = 0)$. It is obtained by solving the following equation:

$$\mathbf{p}_\infty(\tau = 0) = \mathbf{M}_T \mathbf{p}_\infty(\tau = 0), \tag{6}$$

and the existence of such an eigenvector is guaranteed because both the $\mathbf{M}^{+}$ and the $\mathbf{M}^{-}$ are stochastic. The steady-state distribution for general τ is also obtained in a straightforward way. In practice, Eq. (6) can be solved only for $L \lesssim O(10)$ because the size of $\mathbf{M}$ grows as an exponential function of L. From a computational point of view, it is often more efficient to sample configurations by using a Monte Carlo method. Figure 1 demonstrates that the Monte Carlo sampling precisely reproduces the result from Eq. (6). Our Monte Carlo result also shows that the transition-matrix calculation for $L = 10$ is quite accurate in estimating the average density of domain walls in a larger system (Fig. 2). It implies the following: Suppose that we randomly take ten consecutive spins in a large system many times and count the frequency of an arbitrary spin configuration i. Our observation suggests that it will be more or less similar to p_i obtained from the transition-matrix calculation, and it is supported by Monte Carlo calculations (not shown). If a large system can be approximated as a collection of small ones of $L \sim O(10)$, it is because the characteristic length scale is

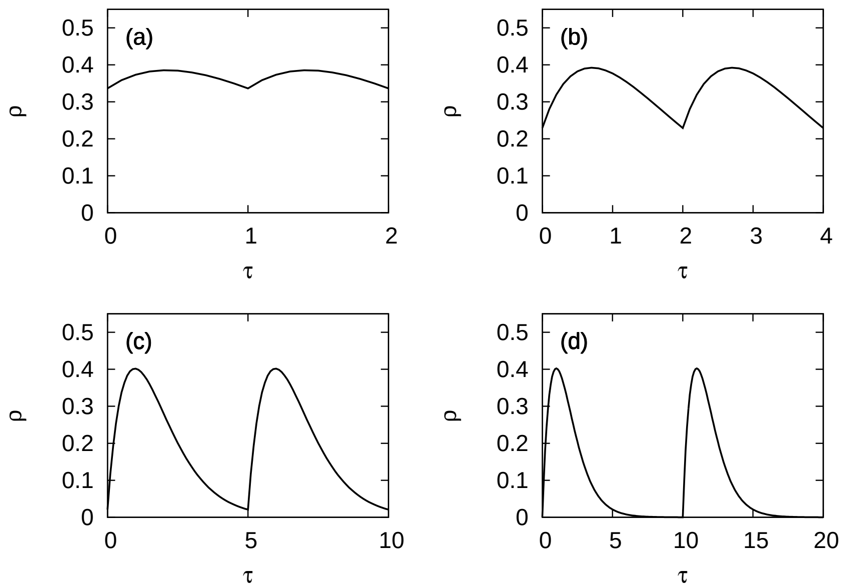

FIG. 1. Density of domain walls in the zero-temperature Ising chain of length $L = 10$, entrained by the field in Eq. (3) with (a) $T = 1$, (b) $T = 2$, (c) $T = 5$, and (d) $T = 10$. Each panel shows numerically exact results from Eq. (6) and Monte Carlo results averaged over 10^5 periods, although they are indistinguishable in this plot.

shorter than $O(10)$. In other words, this observation suggests weakness of the interaction between domain walls. This remark will also be supported by other observations below.

Another important question in this context is whether a dynamic phase transition (DPT) occurs as the half period T is varied. For example, for dimensions higher than one, the Glauber-Ising model undergoes a symmetry-breaking DPT at a sufficiently low temperature as T decreases [19, 20]. Such a DPT is explained by the competition between internal and external time scales for relaxation and driving, respectively. However, such a DPT seems unlikely in our Ising chain, although the temperature is zero: One clue in Figs. 1 and 2 is that the response to $+H_0$ $(0 \leqslant \tau < T)$ is indistinguishable from the one to $-H_0$ $(T \leqslant \tau < 2T)$ for any value of T. The magnetization $m = L^{-1} \sum_i S_i$ also oscillates around zero with preserving the up-down symmetry for any T (not shown). We will present a more quantitative argument for the absence of a DPT by using correlation functions, which we introduce below.

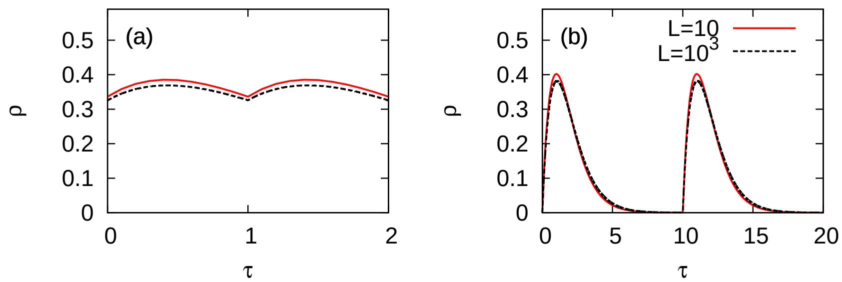

FIG. 2. (Color online) Size dependence in the density of domain walls for (a) $T = 1$ and (b) $T = 10$, obtained by using Monte Carlo calculations, where the solid (dotted) lines represent $L = 10$ ($L = 10^3$).

III. RESULTS

A. Correlation functions

We begin by considering slow driving, e.g., as in Fig. 1(d). One can easily understand the behavior of the density of domain walls: At $\tau = 0$, for example, the field abruptly changes from $-H_0$ to $+H_0$, whereas most of the spins are pointing downward. The density of domain walls thus increases when τ is small. As τ grows further, however, it is followed by a downturn in the density because almost all the spins are aligned in the field direction. Then, the field changes to $-H_0$ again, and all the processes of creation and annihilation of domain walls are repeated anew. We will put this description on a more quantitative ground by considering correlation functions, and then move on to the case of fast driving.

Let us recap the time evolution of an individual spin i during Δt as follows:

$$S_i(t+\Delta t) = \begin{cases} S_i(t) & \text{with probability} \quad 1 - W_i \Delta t, \\ -S_i(t) & \text{with probability} \quad W_i \Delta t, \end{cases} \tag{7}$$

where W_i is given in Eq. (2). In the limit of $\Delta t \to 0$, the time derivative of magnetization and that of the two-point correlation function can be written as

$$\frac{d\langle S_i \rangle}{dt} = -2 \langle S_i W_i \rangle , \tag{8}$$

and

$$\frac{d\langle S_i S_{i+r} \rangle}{dt} = -2 \langle S_i S_{i+r} (W_i + W_{i+r}) \rangle , \tag{9}$$

respectively, where $\langle \cdots \rangle$ means the average over configurations. We now suppose that the system experiences $+H_0$. Enumerating all the possible spin triplets, we can summarize the Glauber transition rates in Eq. (2) as follows:

$$W_i = \frac{1}{2}\left[g_i + (1-g_i)(1-S_i)\right], \tag{10}$$

with $g_i \equiv \frac{1}{4}(1-S_{i-1})(1-S_{i+1})$. By substituting Eq. (10) with Eqs. (8) and (9), we find that

$$\frac{dm(t)}{dt} = \frac{1}{4}\left(3 - 2m - C_2\right), \tag{11a}$$

$$\frac{dC_r(t)}{dt} = \frac{1}{2}\left(3m - 4C_r + C_{r-1} + C_{r+1} - C_{r-1,2}\right), \tag{11b}$$

where $m \equiv \langle S_i \rangle$, $C_r \equiv \langle S_i S_{i+r} \rangle$, and $C_{l,r} \equiv \langle S_{i-l} S_i S_{i+r} \rangle = C_{r,l}$. Note that we have assumed invariance under translation and reflection in the correlation functions. We could also write down the evolution of the three-point correlation functions, but it is already obvious that the equations will not be closed. To proceed, we need to truncate the endless sequence of equations. Our minimalist description is neglecting correlation over a distance greater than two, so it reads as

$$\frac{dm(t)}{dt} = \frac{1}{4}\left(3 - 2m - C_2\right), \tag{12a}$$

$$\frac{dC_1(t)}{dt} = \frac{1}{2}\left(3m - 4C_1 + 1 + C_2 - m\right), \tag{12b}$$

$$\frac{dC_2(t)}{dt} = \frac{1}{2}\left(3m - 4C_2 + C_1\right), \tag{12c}$$

where we have included the evolution of C_2, which appears in Eq. (11a). This description is minimalist in the following sense: Suppose that C_2 is also neglected. Considering Eq. (12a), we see that this makes the evolution of m independent of other correlation functions. Unregulated by higher-order correlations, it has a fixed point at $m = \frac{3}{2}$, which is unphysical. We thus conclude that we need to take into account C_2 at least. Note that our simplified dynamics still admits a fully ordered state with $m = C_1 = C_2 = 1$ as a stationary solution for the static field $H(t) = +H_0$. Calculating the density of domains walls, $\rho(t) = \frac{1}{2}\left[1 - C_1(t)\right]$, we find a striking agreement between Monte Carlo results and Eq. (12), numerically integrated from an initial condition with $m = -1$ and $C_1 = C_2 = 1$ [Fig. 3(a)]. This agreement is also consistent with the remark in the previous section that the correlation length is not greater than $O(10)$.

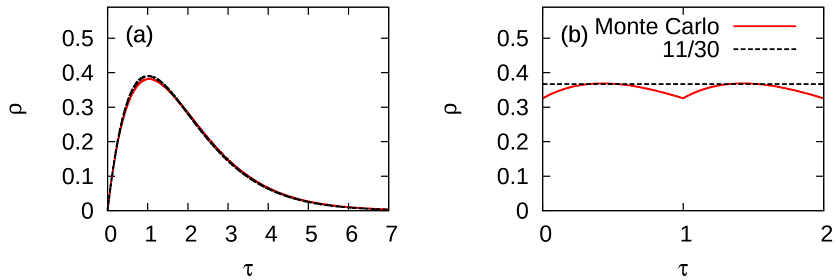

FIG. 3. (Color online) Density of domain walls as a function of τ. (a) The solid lines are obtained by using Monte Carlo simulations for $L = 10^3$ and $T = 15$, whereas the dotted lines are obtained by numerically integrating Eq. (12). (b) Monte Carlo results for $L = 10^3$ and $T = 1$ (solid line) remain close to the value $\frac{11}{30}$ estimated from Eq. (12) in the limit of $T \to 0$ (dotted line).

Having checked our description for slow driving, we may now consider the opposite limit of $T \to 0$. We assume that this limit restores the up-down symmetry so that m is negligible in Eqs. (12b) and (12c) in an average sense. This assumption is supported by the following argument: If any remnant magnetization $m \neq 0$ exists, it means that the system is unable to respond to such a rapid field modulation. According to this idea, C_2 would not change appreciably upon the field reversal, either. When $H(t) = -H_0$, Eq. (12a) takes a slightly different form:

$$\frac{dm(t)}{dt} = \frac{1}{4}\left(-3 - 2m + C_2\right), \tag{13}$$

where the right-hand side is written in terms of the same correlation functions based on the ”freezing” scenario above. The summation of Eqs. (12a) and (13) expresses the total change in m during one period, which must vanish in a steady state. This immediately leads to the conclusion that $m = 0$. In this way, we can argue that a symmetry-breaking DPT, as a result of the competition between relaxation and driving time scales, should be absent in our system. Put differently, the relaxational time scale does not grow longer than the one for driving, and this is consistent with our observation of short correlation lengths. As a side remark, we add that the statement of vanishing m contains a subtle point: If m was strictly zero all the time, it would imply $\frac{dm}{dt} \neq 0$ in Eq. (12a) or (13), which is self-contradictory. A correct explanation is rather that m will keep changing around zero with a small magnitude. In addition to $m = 0$ in an average sense, the steady-state condition requires that both $\frac{dC_1}{dt}$ and $\frac{dC_2}{dt}$ must also vanish. Solving the set of linear equations resulting from Eqs. (12b)

and (12c), we estimate the stationary density of domain walls as $\rho = \frac{1}{2}(1 - C_1) = \frac{11}{30}$. This calculation agrees well with our Monte Carlo result, confirming the existence of domain walls in the presence of a fast switching field [Fig. 3(b)].

B. Domain statistics

So far, we have focused on the lowest-order ones among the infinite hierarchy of correlation functions, and this turns out to be enough to describe certain average quantities, such as the density of domain walls. Now, let us proceed to the detailed statistics of domains to gain more information. We begin by considering how domains evolve in time when the field is taken to be $+H_0$. Let P_n denote the density of down-spin domains of length n so that $\sum_n nP_n$ is equal to the fraction of down spins. We have four mechanisms that affect P_n.

(1) A domain of length n disappears when any of its down spins flips upward. According to Eq. (2), two spins at the boundary flip with rate 1, whereas the rate is reduced to $\frac{1}{2}$ for the other $(n-2)$ spins in the bulk. Therefore, the total rate of loss amounts to $1 \times 2 + \frac{1}{2} \times (n-2) = \left(\frac{n}{2} + 1\right)$, multiplied by P_n, for $n \geq 2$. Note that this formula does not cover the case of a single-spin domain, which disappears via $\{\cdots \uparrow\downarrow\uparrow \cdots\} \Longrightarrow \{\cdots \uparrow\uparrow\uparrow \cdots\}$ with rate 1.

(2) The density P_n increases when a domain of length $n+1$ shrinks by one at the boundary. The contribution is counted as $2P_{n+1}$ because of the two boundary spins.

(3) We can increase P_n by dividing a domain of length $l \geq n+2$ into two pieces in such a way that

$$\{\cdots \uparrow \underbrace{\downarrow \cdots \downarrow\downarrow\downarrow \cdots \downarrow}_{l} \uparrow \cdots\} \Longrightarrow \begin{cases} \{\cdots \uparrow \underbrace{\downarrow \cdots \downarrow}_{n} \uparrow \underbrace{\downarrow \cdots \downarrow}_{l-n-1} \uparrow \cdots\} & \text{with rate } \frac{1}{2}, \\ \{\cdots \uparrow \underbrace{\downarrow \cdots \downarrow}_{l-n-1} \uparrow \underbrace{\downarrow \cdots \downarrow}_{n} \uparrow \cdots\} & \text{with rate } \frac{1}{2}. \end{cases} \tag{14}$$

If $n \neq l-n-1$, this has two different possibilities, each with rate $\frac{1}{2}$, so the contribution to P_n from the domain of length l is equal to P_l. Even if $n = l-n-1$, the contribution is still P_l because the division creates two domains of length n with rate $\frac{1}{2}$. In total, this third mechanism contributes $\sum_{l=n+2}^{\infty} P_l$ to P_n.

(4) The last mechanism is to merge a domain of size $l \leq n-2$ and another with size $n-l-1$ to create a domain of size n. We can visualize it as

$$\{\cdots\uparrow\underbrace{\downarrow\cdots\downarrow}_{l}\uparrow\underbrace{\downarrow\cdots\downarrow}_{n-l-1}\uparrow\cdots\} \Longrightarrow \{\cdots\uparrow\underbrace{\downarrow\cdots\downarrow\downarrow\downarrow\cdots\downarrow}_{n}\uparrow\cdots\} \text{ with rate } \frac{1}{2}. \tag{15}$$

To evaluate the probability of this event, we need to know the probability of the configuration on the left-hand side. The independent-interval approximation (IIA) suggests that the lengths can be regarded as totally uncorrelated so that the probability can be expressed as $P_l P_{n-l-1}$ [21]. The total contribution of this mechanism is thus approximately written as $\frac{1}{2}\sum_{l=1}^{n-2} P_l P_{n-l-1}$.

Gathering all these terms, we arrive at

$$\frac{dP_1}{dt} = -P_1 + 2P_2 + \sum_{l=3}^{\infty} P_l, \tag{16a}$$

$$\frac{dP_n}{dt} \simeq -\left(1+\frac{n}{2}\right)P_n + 2P_{n+1} + \sum_{l=n+2}^{\infty} P_l + \frac{1}{2}\sum_{l=1}^{n-2} P_l P_{n-l-1} \text{ for } n \geq 2. \tag{16b}$$

The next step is to consider the dynamics of up-spin domains with keeping the same field direction. Similar to P_n, we define Q_n as the density of up-spin domains of length n. We have five mechanisms to affect Q_n.

(i) In the first mechanism, a domain of a single up spin evaporates via $\{\cdots\downarrow\uparrow\downarrow\cdots\} \Longrightarrow \{\cdots\downarrow\downarrow\downarrow\cdots\}$ with rate $\frac{1}{2}$. This takes place only for $n=1$.

(ii) Again, this second mechanism applies only to $n=1$. A domain of length 1 can be created via $\{\cdots\downarrow\downarrow\downarrow\cdots\} \Longrightarrow \{\cdots\downarrow\uparrow\downarrow\cdots\}$ with rate $\frac{1}{2}$. For this to happen, we have to pick up a down spin surrounded by two other down spins. For a down-spin domain of size l, we have $l-2$ such spins. Therefore, we compute this contribution as $\frac{1}{2}\sum_{l=3}^{\infty}(l-2)P_l$. Note that the dynamics of Q_n is coupled to that of P_n.

(iii) The third mechanism describes a loss due to the growth from length n. The domain can grow to the left or right, each with rate 1, so the contribution becomes $-2Q_n$.

(iv) A domain of length n can be gained from the growth process as well, when a domain of length $n-1$ expands to n by flipping a spin upward at the boundary with rate 1. However, we cannot simply write it as $2Q_{n-1}$ because the spin flip may merge this

domain with another. For example, if we look at the left boundary, the following process creates a domain of length n:

$$\{\cdots \Downarrow\downarrow \underbrace{\uparrow\cdots\uparrow}_{n-1}\downarrow\cdots\} \Longrightarrow \{\cdots\Downarrow\underbrace{\uparrow\uparrow\cdots\uparrow}_{n}\downarrow\cdots\}, \tag{17}$$

whereas the following does not:

$$\{\cdots \Uparrow\downarrow \underbrace{\uparrow\cdots\uparrow}_{n-1}\downarrow\cdots\} \Longrightarrow \{\cdots\Uparrow\underbrace{\uparrow\uparrow\cdots\uparrow}_{n}\downarrow\cdots\}. \tag{18}$$

In short, it depends on the direction of the spin drawn as a double arrow on the leftmost side. In a similar spirit to the IIA, we assume a well-defined probability Φ for the spin to point downward so that the contribution becomes $2Q_{n-1}\Phi$.

(v) The last mechanism is to merge two up-spin domains, one with size l and the other with size $n-l-1$ as follows:

$$\{\cdots\downarrow\underbrace{\uparrow\cdots\uparrow}_{l}\downarrow\underbrace{\uparrow\cdots\uparrow}_{n-l-1}\downarrow\cdots\} \Longrightarrow \{\cdots\downarrow\underbrace{\uparrow\cdots\uparrow\uparrow\uparrow\cdots\uparrow}_{n}\downarrow\cdots\} \text{ with rate 1.} \tag{19}$$

As before, we resort to the IIA to estimate the contribution as $\sum_{l=1}^{n-2} Q_l Q_{n-l-1}$.

To sum up, we have derived equations for Q_n as

$$\frac{dQ_1}{dt} = -\frac{1}{2}Q_1 - 2Q_1 + \frac{1}{2}\sum_{l=3}^{\infty}(l-2)P_l, \tag{20a}$$

$$\frac{dQ_n}{dt} \simeq -2Q_n + 2Q_{n-1}\Phi + \sum_{l=1}^{n-2} Q_l Q_{n-l-1} \text{ for } n \geq 2. \tag{20b}$$

Even if $H(t) = -H_0$, we can derive essentially the same as Eqs. (16) and (20), provided that the variable Q_n indicates domains in the direction of the field, whereas P_n does in the opposite direction.

Suppose that T is so short that the down-spin domains are effectively subjected to both Eqs. (16) and (20). The steady-state condition implies that $\frac{dP_n}{dt} + \frac{dQ_n}{dt} = 0$ for every $n \geq 1$. As the up-down symmetry is restored, we may also equate every Q_n with P_n with setting $\Phi = \frac{1}{2}$. We finally end up with the following set of equations:

$$0 = -\frac{7}{2}P_1 + 2P_2 + \sum_{l=3}^{\infty}\frac{l}{2}P_l, \tag{21a}$$

$$0 = P_{n-1} - \left(3 + \frac{n}{2}\right)P_n + 2P_{n+1} + \sum_{l=n+2}^{\infty} P_l + \frac{3}{2}\sum_{l=1}^{n-2} P_l P_{n-l-1} \text{ for } n \geq 2. \tag{21b}$$

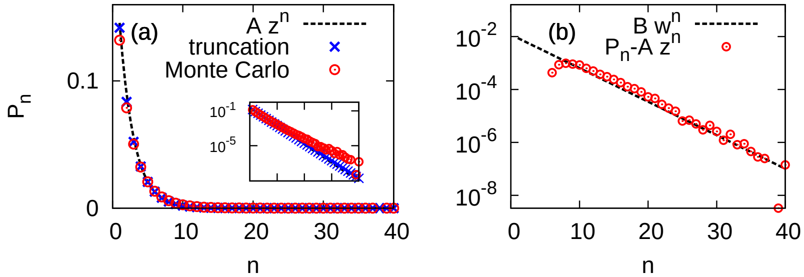

FIG. 4. (Color online) Domain size distribution. (a) The dotted line shows the trial solution $P_n = Az^n$ with $A \simeq 0.236\,629$ and $z \simeq 0.615\,633$ that approximately solves Eq. (21). The crosses represent a numerically exact solution of Eq. (21) truncated at $n = 50$. The circles are Monte Carlo results for the Ising chain of length $L = 10^4$. The inset shows the same data in a semi-logarithmic plot. (b) Correction from the simple exponential form [Eq. (22)]. We estimate $B \simeq 0.01$ and $w \simeq 0.75$ by fitting the data on a logarithmic scale.

Our trial solution is an exponential distribution, i.e., $P_n = Az^n$ with positive constants A and $z < 1$. Substituting this into Eq. (21a), we obtain $z \simeq 0.615\,633$. It is worth noting that z would be equal to $\frac{1}{2}$ if all the correlations were destroyed as in the infinite-temperature limit. Although this trial solution does not exactly solve Eq. (21b), we can estimate the amplitude $A \simeq 0.236\,629$ by taking $n \to \infty$. As a cross-check, we truncate Eq. (21) by setting $P_n = 0$ for $n > 50$, and solve the 50 coupled equations for $P_1, \ldots, P_{50}$ simultaneously. It confirms the validity of our trial solution even for small values of n as shown in Fig. 4(a). Of course, we have to ask ourselves whether Eq. (21), involved with several uncontrolled approximations, correctly describes the domain dynamics. This is checked by simulating an Ising chain of length $L = 10^4$ to sample the domain length distribution. As depicted in Fig. 4(a), the result shows that Eq. (21) works qualitatively but tends to underestimate P_n when n is large. The correction from Az^n reveals another length scale in the following form:

$$P_n - Az^n \simeq Bw^n, \tag{22}$$

with $B \simeq 0.01$ and $w \simeq 0.75$ [Fig. 4(b)]. The second length scale corresponds to roughly four lattice spacings, about twice larger than the first one, but its the origin is not fully understood yet.

IV. SUMMARY

To summarize, we have considered the zero-temperature Glauber dynamics in the 1D Ising chain driven by rectangular pulses of period $2T$ and strength equal to J. We have argued that the driving interrupts the coarsening so that the density of domain walls converges to a nonzero stationary value $\simeq \frac{1}{3}$ in the limit of fast driving. We have also calculated the steady-state distribution of domain lengths in the same limit by using the IIA, and the result indicates the existence of finite nontrivial correlation. Moreover, our Monte Carlo calculation shows that the actual density is higher than expected from simple exponential decay, revealing the existence of the second length scale, about twice larger than the first one.

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